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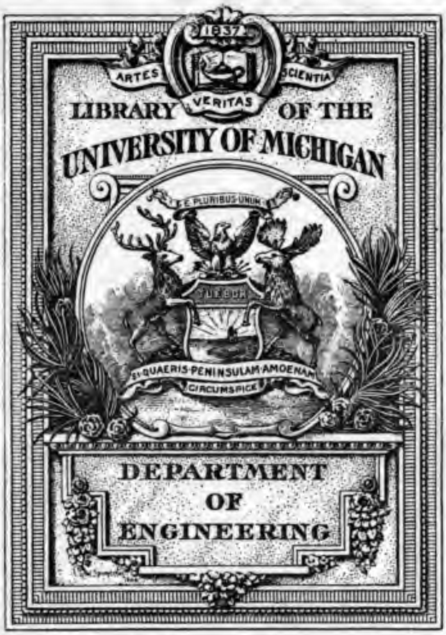
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THE ELASTIC ARCH

WITH SPECIAL REFERENCE
TO THE
RE-ENFORCED CONCRETE ARCH

BY

BURTON R.utherford LEFFLER
Engineer of Bridges, Lake Shore and Michigan Southern Railway
Assoc. M. Am. Soc. C.E.

NEW YORK
HENRY HOLT AND COMPANY
1906

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PREFACE.

THIS work is an attempt at the exposition of a difficult subject.

The introduction of reenforced concrete construction has made it necessary to evolve some practicable and fairly easy method of designing arches.

The writer was direly in need of a method, for in his hands has been placed the designing of many arches during the past few years.

To Prof. H. T. Eddy and Prof. Wm. Cain, I am indebted for the application of the equilibrium polygon to arches ; to Prof. Irving P. Church for his clear exposition of Eddy's method ; and to Prof. M. A. Howe for the effect of axial thrust.

I lay claim to several new features. I have shown how the subject can be deduced from one simple equation. The easy method of drawing the closing line of the equilibrium polygon was given by me in the Transactions, Am. Soc. C. E., vol. lv. Prof. Howe's results of axial thrust, I have applied graphically.

A correct and simple method of designing a reenforced concrete section, for combined thrust and moment, is given ; and it is the first presentation as far as I know.

176687

A graphical analysis of an arch for oblique forces is the first, I believe, to be presented. I am indebted to Prof. Wm. Cain for a hint in this.

Beside the above writers, I feel under no obligations to any one. In fact, none of the writers presents a method fit for the drafting-room.

The writer made the following study while holding the position of chief draftsman; and hence, believes he can speak with authority concerning the difficulties of the subject.

CLEVELAND, O.,
DECEMBER, 1906.

ERRATA.

Page 7, third line from bottom, read $s = \frac{T}{A} + \frac{MC}{I}$

Page 29, line 7, for $T\frac{dl}{E}$ read $\frac{T}{A}\frac{dl}{E}$

HOW TO STUDY.

READER, permit me to make a few suggestions concerning an art which all assume to know—the art of mentally grasping a subject.

All structural draftsmen know the value of a general drawing as a preliminary to detailing. The erection diagram, and small skeleton outline of the whole structure shown on each detail sheet, are very important for an easy grasping of the subjects.

We must obey an important psychological law. It is this. *The mind, in grasping any subject, must be allowed to see it, first, as a vague whole.* The vague whole being first seen, further study reveals the details. Synthesis and analysis must go together, but synthesis must come first. It is this law that compels the engineer in the field to first make a reconnaissance and preliminary survey before locating his line.

Superiors in office, giving instructions to their subordinates, are often grossly in the wrong in this matter. They carry a subject for quite a while in their minds, and then present it all at once to their subordinates, expecting them to see it in an instant as they themselves see it.

Writers on the arch have invariably neglected to give a rough view. I have attempted to overcome this by giving a preliminary view of arch action, and then a definite statement of this action. Further exposition is nothing but a detailed account of the definite statement.

If you, reader, have had the right preliminary training, the following pages should be easy reading.

In making the analysis of a subject, each part composing the subject must be examined as if it stood alone. The separate results are then combined. For instance, in mechanics the effect of simultaneous forces is gotten by taking the forces as if they acted in succession and then combining the respective effects. This enables us to study each force alone. This same mental device is used in all departments of science. If its use were made explicit it would make clear many hard subjects.

In the following pages you will find this device used. I refer to the two terms composing Eq. 1 in Art. 3. The effect of each term is considered as if it acted alone.

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art for *k* line ravity axis		
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table, see Art. 4 of text.

PART I.

THE ANALYSIS OF AN ARCH.

PART I.

THE ANALYSIS OF AN ARCH.

Introduction. Art. 1.

In the following pages, the writer wishes to simplify the theory of the elastic arch as evolved by Prof. H. T. Eddy and Prof. Wm. Cain.

The proportioning of steel for any section will be shown. Scattered throughout the pages will be found various comments pertaining to the subject. In the appendix, some matters of a controversial nature will be found.

The steel will be considered as taking tension only, and concrete compression only. The arch will be taken as a plain concrete one, and the conditions determined under which steel must be used.

Few designers have the time or ability to understand all of the fine points in the elastic theory, but the application of this theory by graphics is so simple that any draftsman, ignorant of the theory, can easily apply the method. This is one of the peculiar advantages of graphics.* The writer has repeatedly taught draftsmen the method, but not necessarily the logic of it.

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* For some of the educational defects of graphics. see Eng. News of Aug. 31, 1905, and also the appendix.

The reader is supposed to have a knowledge of elementary mathematics, the elements of mechanics of materials, especially that part pertaining to eccentric stresses, and the theory of the equilibrium polygon. In some of the deductions a little calculus will be used, but the reader can ignore this, taking the result on trust. There is really nothing abstruse, at least for the ordinary symmetric arch.

A Rough View of Arch Action. Art. 2.

Let Fig. 2, Pl. 4, represent an arch. For our present purpose, it is not necessary to consider the loading. Whatever the loading, the arch will bend at the various sections, and exert a push on the abutments. Now let us consider the internal forces acting at any section, *C*, *D*, of the arch ring. Since the arch ring bends and exerts a push on the abutments, the internal forces, at the section, can be divided into two parts; namely, a couple which resists the bending, the same as in a beam, and a single force. At some sections, and for certain loading, the couple may equal zero. At such sections there is no bending, but the single force always acts.

Notice that in a simple beam, the single force appears as shear. This is because the simple beam exerts no push, lengthwise, on its abutments. Notice, too, that in a simple beam, the bending is always in one direction. In an arch, it is in both directions.

The division of the internal forces into two parts can also be proved by the laws of static equilibrium, as is usually done in treatises on the arch.

It is of prime importance that the reader should grasp the idea of the division of the internal forces at any section, as outlined above. The whole theory of the elastic arch can be derived from this idea. After presenting the graphic solution, an effort will be made to do this.

The single force and couple, acting at any section, can be reduced to a single force.* These final single forces, acting at successive sections, make up the pressure curve. The primary problem is to find the pressure curve. Since, by graphic processes, it is necessary to take the successive sections a short finite distance apart, the pressure curve will be a polygon approximating the true curve.

Meaning of the Pressure Curve. Art. 3.

Suppose, for the time being, that the pressure curve has been located, and let us examine into its exact meaning at any section, *CD*, Fig. 2. Conceive the arch ring cut into two parts at the section, and one part shown in Fig. 3. It is evident that each part is now a free rigid body. The single internal force, acting on the section and represented by the corresponding part of the pressure curve, holds in equilibrium the remaining forces acting on the part.

---

* Bowser's Analytical Mechanics, 6th Ed., Art. 54.

The remaining forces are the external loads and the abutment push. In Art. 5 will be found further use of the pressure curve.

The following symbols will now be defined.

Let R=the magnitude of the single internal force.

Let M=the internal moment, being that of the couple referred to in the previous pages.

Let s=unit stress on any fiber, distant c from the gravity axis. The gravity axis is often called, improperly, the neutral axis.

A=area of the section, and I its moment of inertia.

E=modulus of elasticity of the material.

l=the length of the gravity axis measured, say, from left springing.

dl=a short finite distance between any two successive sections for the graphic process, but an infinitesimal for theoretic demonstrations.

x and y are the coordinates of the gravity axis at the section.

e=the radial distance from the gravity axis to the point of application of R.

By a section is meant one at right angles to the gravity axis.

R is resolved into two components, U, parallel to the section, called the shear; and T, at right angles, called the thrust. In most arches, U is small and hence may be neglected.

At this stage of our study, we will explain, roughly, the manner of presenting the remainder of the subject. Thus far the reader has encountered nothing of an abstruse nature. The division of the subject will be into four parts. In the first part, will be given, dogmatically, the conditions underlying the simple graphic process and its application to a symmetric arch with vertical loads. In the second part, the mathematical reasoning underlying the subject will be given. In the third part, the analysis of an unsymmetric arch will be given. This is the order of simplicity. Any one desiring to omit the mathematics, can do so. The first part will cover most cases of design, and represents the instruction given by the writer to his draftsmen. The fourth part is the appendix.

Referring to Fig. 3 and the symbols, and neglecting U, we have from mechanics of materials,

$$S=\frac{T}{A}+\frac{Mc}{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Eq. 1 is a mathematical statement of the previous rough view of arch action. Notice, $Te=M$, also since the external loads are vertical, the horizontal component, H of R, is the same for all sections. Let $t=$ the vertical distance from the gravity axis to the line of action of R, then $M=Ht$.

Eq. 1 is of prime importance. As previously stated, it underlies the whole arch theory; and in Part II it will be shown to be fundamental.

We will now state, dogmatically, the final conditions for a hingeless elastic arch.

$$\Sigma\frac{Mdl}{EI}=0,\quad \Sigma\frac{Mydl}{EI}=0,\quad \text{and}\quad \Sigma\frac{Mxdl}{EI}=0.$$

In these conditions, by properly taking the dimensions of the arch ring, $\frac{dl}{I}$ can be made a constant. This will be shown later. Remember, now, that dl means a short finite length for graphical purposes. Substituting the value of M, the conditions can be written,

$$\frac{Hdl}{EI}\,\Sigma t=0,\quad \frac{Hdl}{EI}\,\Sigma tx=0,\quad \text{and}\quad \frac{Hdl}{EI}\Sigma ty=0.$$

For any properly taken ring, the fraction is a constant; and as far as the graphical requirements are concerned, it can be omitted. The conditions can be written,

$$\Sigma t=0 \cdot\cdot\cdot\cdot (2),\quad \Sigma tx=0 \cdot\cdot\cdot (3),\quad \Sigma ty=0 \cdot\cdot\cdot (4).$$

These conditions are the very foundations of the graphic method. They are deducible from Eq. 1 by neglecting $\frac{T}{A}$ as will be shown in Part II. Further on, it will be shown how to bring in $\frac{T}{A}$.

Notice, carefully, the exact meaning of the conditions. The gravity axis must be divided into short lengths, dl; the ordinates, x and y, at the center of each length, measured; t measured at the center of each length; the products, tx and ty, found for each length, and finally the respective summations made for the whole gravity axis.

Right here it might be well to call attention to the value of regarding E, the modulus of elasticity, as a constant. Strenuous efforts have been made during the past few years to prove it a variable for working stresses. For all practical cases of design, it is a constant.*

Location of the Pressure Curve. Art. 4.

The problem is to locate the pressure curve so as to fulfil conditions (2), (3) and (4). This will be illustrated by taking an actual example as worked out in the drafting room under the writer's direction.

See Pl. 2. Span of intrados curve is 33 ft. Rise, 15 ft. Thickness at crown, 2 ft.

As far as the analysis is concerned, *oy* is the real span; *o* and *y* being points on the gravity axis at which the abutment tangents to gravity axis are supposed to be rigidly fixed in direction. Practically, this means where the arch ring joins the abutments by a somewhat sudden enlargement of section.

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* See Concrete, Plain and Reenforced, by Taylor and Thompson, p. 286.

The loads for the arch were very heavy, being Cooper's *E*60 with impact added.

In designing an arch, the intrados curve and crown thickness must be assumed. The span is then divided into a number of equal short spaces. The division is made on the dotted ordinates with an ordinate at the center. This determines $\frac{dl}{I}$ at the crown. The remainder of the ring must be taken so that $\frac{dl}{I}$ has the crown value for each division. Remember that dl is measured along the gravity axis which is yet undetermined. For a trial value, dl may be taken along the intrados. I is taken at the center of each division of gravity axis.

For a circular arch, the determination of the section near the abutment is sufficient. Three points of the gravity axis are thus found, and the curve drawn. Several trials are necessary to determine the sections.

The taking of $\frac{dl}{I}$ a constant, is an artifice to simplify the analysis.

In making the division, the writer neglects the moment of inertia of the bars. This is a slight error. It is customary to take the arch ring twelve inches long, perpendicularly to the paper. The section of the arch ring is, then, a rectangle twelve inches wide and of a variable depth, h, say. $\frac{dl}{I}$ then becomes $\frac{dl}{h^3}$. By means of a table of cubes, the section can be determined very quickly.

The loads having been determined, a load line is laid off, and a trial equilibrium polygon drawn as shown below. Notice, carefully, the system of numbering the ordinates. For instance, ray 2, in the lower part of the force diagram, crosses ordinate 2 on the left hand side of the arch. Each load is designated by the numbers between which it occurs.

Remember now that we are simply stating the method. The reasoning will be given further on. The student should familiarize himself with the actual solution of an arch by taking an example on the drafting board. In this case, art before science.

Our immediate problem is to draw a line, $m\ m_1$, so that the sum of the ordinates, called m ordinates, from this line to the trial equilibrium polygon, shall be zero. Ordinates measured above are plus, and below, minus. Also, the center of gravity of these ordinates, treated as forces acting down, say, must lie in the same vertical line as the center of gravity of the ordinates within the trial polygon.

Let us inspect Table No. 1. Col. 1 contains the numbers of the ordinates. Columns 2 and 3 contain those parts of the ordinates included in the trial polygon, measured to the *scale used for the arch ring.* Col. 4 contains the respective differences of symmetric ordinates about the center. Having these figures, use them in the formula shown below the force diagram, for computing vw. vw always occurs on that side of the trial polygon having the longer ordinates, the left in this case. The middle ordinate to $m m_1$, is found by dividing R, the sum of the ordinates within the polygon, by the number of the ordinates, or number of spaces plus one. In the figure the number of spaces is 16, ordinates 17.

Likewise in the arch, measure the ordinates between $o\ y$ and gravity axis. These values are found in column 8. Divide the sum of the ordinates by the number, or by the number of spaces plus one. This number is the same as for line $m\ m_1$. Draw a line, $k\ k_1$, parallel to $o\ y$ and at a distance above it equal to the quotient just found. Ordinates, called k ordinates in value, measured from the line $k\ k_1$, to the gravity axis of the arch, have a similar relation to the ordinates within the arch as the m ordinates to the line $m\ m_1$, have to the ordinates within the trial polygon.

The line $m\ m_1$, cuts the trial equilibrium polygon in points E and E_1. Project these points upwards to e and e_1 in line $k\ k_1$. Points e and e_1 are points in the required pressure curve.

The m and k ordinates are found in columns 6, 7, 9 and 10. The respective algebraic sums should be zero. Of course, by graphics there will be slight errors.

Next multiply each m and k ordinate by its corresponding y ordinate, due regard being paid to signs. These products with their algebraic sums are found in columns 11 to 14 inclusive.

Referring to the force diagram, draw P_1r parallel to $m\ m_1$. From r draw an indefinite horizontal line. The true pole for the pressure curve lies on this line. A pole taken anywhere on this line will give an equilibrium curve passing through the points e_1 and e. Why? The pole distance for the pressure curve

$$=\Sigma \frac{my}{\Sigma ky}(\text{Trial Pole Dist.}) \text{ as shown below table.}$$

Further on the reasoning underlying this will be shown. Having the true pole, begin at e or e_1 with the proper ray and draw the true pressure curve. Beginning at either point, the curve should pass through the other point.

Determination of Unit Stresses. Art. 5.

The pressure curve being located, the unit stresses at the various sections can be found. The sections should be taken at the points where the dotted ordinates cut the gravity axis, and not at the loads.

As an example take a section at ordinate 2 on the left hand side. Ray 2 in lower part of force diagram shows the magnitude of the resultant acting at the section. As explained in Art. 3, this is divided into two components, a shear and direct thrust. The magnitude of the thrust is 42,000 lbs. The shear, being small, is neglected.

Here is the important point in re-enforced concrete construction. *Wherever the pressure curve falls without the middle third of the plain concrete section, tension occurs in the section. Hence, steel is required only at such sections.* We are assuming that steel only takes tension, and concrete compression. This is now considered the best practice.

Referring, again, to section 2, which is shown by itself in Fig. 4, we will give the correct method of determining the tension in the steel and compression in the concrete. The reader must have clearly in mind the theory of eccentric loads.

The direct thrust, divided by the area of the section and multiplied by two, gives the maximum compression the joint would sustain at the outer edge nearer the thrust—the upper in this case—

were the thrust applied just at the edge of the middle third; but since it is applied beyond, we must determine how much the extra pressure is, and also determine the amount of steel necessary to take the tension at the lower edge. *Were concrete capable of taking tension, the extra pressure would be directly proportional to the distance of the point of application of the thrust beyond the edge of the middle third.*

We will assume that the foregoing law still holds, since the steel is to supply the lack of tension in the concrete. This is a slight error because the elasticity of steel and concrete differ. The law can be restated as follows. *The moment, for extra pressure at the outer edge nearer the thrust, is equal to the distance of the point of application beyond the edge of the middle third multiplied by the thrust.* Use this moment the same as if it occurred in a simple reenforced concrete beam, determining the stress in the concrete and the tension in the steel. Add the stress for the concrete to twice the average compression for the maximum compression in concrete.

It is assumed that the reader is acquainted with the theory of the simple reenforced concrete beam as outlined, say, in "Concrete, Plain and Reenforced," by Taylor and Thompson. Plate 3 represents the drafting-room instructions issued by the writer for designing reenforced concrete beams. The table in this plate is calculated for the several ratios of the moduli of elasticity of steel and concrete. For a mixture of 1 : 3 : 6, the writer uses a ratio of 15.

Attention should be called to fallacious methods of determining the amount of steel at any section. Some parties, interested in selling steel bars, say that the moment equals the eccentricity of the thrust multiplied by its magnitude. The fallacy is evident since this gives a larger moment than the correct method. Here, as in the past history of science, pecuniary interests have obscured a simple theoretic truth.

Tests of simple reenforced concrete beams for ultimate strength have furnished valuable results, but the application of formulas, based on ultimate strength to cases of combined thrust and moment, is practically impossible. At least, the writer has seen no correct application.

It is evident that the writer does not believe in the ultimate strength method of designing, except in cases where theory has not been developed, as, for instance, in continuous slabs, or slabs supported at all edges. But more about this in the appendix.

In the foregoing it was assumed that the center of the bars coincided with one edge of the arch ring, the intrados in the case cited. But, since this is practically impossible, the bars must be placed a short distance within, usually 1½ inches to 3 inches.

The depth of the ring should now be taken from the upper edge to the center of the bar.

We will now give in detail the design of the arch ring at section 2 on the left. Notice, carefully, in Fig. 4, the relation of all the factors entering into the problem.

The depth of the ring to the center of the bars is 22 inches. The area above the center of the bar is 264 sq. inches. The area below is neglected. $(42,000 \div 264) \times 2 = 323$ lbs. This is twice the average compression, and is *part* of the total compression on the outer edge of concrete. The thrust is applied 7 inches above the gravity axis, and hence is outside of the middle third. The width of the middle third is $7\frac{1}{3}$ inches. Hence, the moment arm for the thrust is $7-3\frac{2}{3}=3\frac{1}{3}$ inches. The moment is $42,000 \times 3\frac{1}{3} = 140,000$ inch pounds. Having the section and moment, use formula (2) on Plate 3 for computing the stress in steel, getting 9,000 lbs. per sq. inch. About 0.78 sq. inches of steel is required. Use the stress just found in formula (1), getting 200 lbs. per sq. inch as extra compression in concrete. The total compression on the concrete is $323+200=523$ lbs. per sq. inch.

Thus the stresses can be found at any section. Should the stresses be found too large, or too small, a different section must be assumed. To be exact, a new pressure curve would have to be located, and each section reinvestigated. In actual designing, this is seldom necessary for the reason that an expert designer can nearly always select a proper section at the beginning, at least for ordinary spans.

It is better to select a section that will have stresses a little high, for on thickening the ring it is well known that the stresses will be less than the calculated ones. If too thick a ring is selected and then thinned, it would be necessary to recalculate in order to be sure that the revised section is safe.

The first part of our problem is solved. Notice what has been omitted. Nothing has been said about temperature stresses, or about the effect of $\frac{T}{A}$ in Eq. 1.

The design of the abutments has not been considered; but this is outside of the present problem, since we have tacitly assumed them to be immovable. The design of the abutments really belongs to retaining walls and foundations. Briefly, the abutment must be designed to resist the push of the arch ring, and the overturning effect of the earth. The push is represented by the first and last rays in the pressure curve. To make the design, all of the forces acting on the abutment must be considered. See any standard work on masonry construction.

Time to Calculate an Arch. Art. 6.

A large solid arch should be calculated for four different positions of the live load, at least. A full load, a three-fourths load, a half-load and a one-fourth load should be taken.

Two draftsmen can analyze an arch for the above different loadings, designing the sections and determining the width of the abutments in about 30 hours of total labor. Much depends upon a systematic arrangement of the work.

Shrinkage of Concrete. Art. 7.

This is an unsettled matter. Cracks in the arch ring, parallel to the axis, are not likely to occur. The action of the loading tends to prevent this. Cracks, running across the barrel of the arch and at right angles to the axis, do appear. The writer puts in steel to the amount of $\frac{1}{1200}$ of the area. For most arches he puts in a row of bars having $\frac{1}{4}$ sq. inch area, and running parallel to the axis along the intrados and extrados. These bars are placed two feet apart. So far experience has been our only guide.

Shear. Art. 8.

Many writers seem to think because the shear on a radial section is small, there is no shear on other sections, but this is a mistake. Any block sustaining simple compression has shear along various sections. In fact the block may fail by shearing on an oblique section.

In Fig. 10 take any two vertical sections symmetric about the center of the arch. Then it is evident that the shear on these sections must be equal to the load between the sections. Imagine a cut to be made on these sections. It is evident that the stability of the arch is endangered. This shear, on the supposition that concrete carries it all, should not exceed 150 lbs. per sq. inch for loads including impact effect. In this, of course, the area is taken on the vertical sections, and not radially.

Summary. Art. 9.

The foregoing part of this work covers all of the essentials in the elastic theory which is necessary for the design of ordinary arches. A summary of the leading points will now be made.

Steel for tension is only required where the pressure curve is out of the middle third of the plain concrete section. In actual designing, the writer puts in more than this requirement to avoid much lapping of bars, nicety of placing them, and the use of too many different sizes of bars.

In all concrete designing, the low character of the labor employed in executing the design, must be kept in mind.

Temperature stresses must be taken care of.

In arches flatter than one-fourth the span, the effect of $\frac{T}{A}$ should be included.

Simple shear at any vertical section should be calculated.

Wherever bars are spliced, it should be done by giving them plenty of lap, say about 50 times their least diameter.

The working unit stresses are :

Concrete	in	Compression	500 lbs. per sq. inch.
"	"	Tension	nothing.
"	"	Shear	150 lbs. per sq. inch.
Steel	"	Tension	12000 " per sq. inch.

These stresses are to be used only for external loads including impact. There is impact wherever the roadway is close to the arch ring. The amount of this impact is an unsettled question.

The reader has all of the essentials of arch theory. In the appendix, the writer has taken up some disputed points, and has allowed himself to indulge in some pungent remarks.

PART II.

THEORETIC DEDUCTIONS.

PART II.

THEORETIC DEDUCTIONS.

Approximations Necessary for the Use of Eq. 1. Art. 10.

Now comes the part of our task that most readers will consider difficult. But let us stick close to our rough view of arch action, and its definite statement as given in Eq. 1.

Before using Eq. 1, let us examine into an apparently simple case of direct compression. In Fig. 6, let P be an axial load applied perpendicularly to a right section, MN, of a short rectangular block.

Then, according to the elastic theory, for moderate values of P, the block will be uniformly shortened. Very simple, you say. But suppose the section, MN, is an oblique one as in Fig. 6*a*. It is evident, since the fibers are not of the same length, that the shortening on each fiber cannot be the same; or if this shortening is the same, the stresses acting on the fibers are unequal. Whichever way it is viewed, it is now evident that we are confronted by a problem of considerable difficulty.

This is largely the very gist of the difficulty in the elastic theory of the arch. In the arch no two right sections are parallel. So great is this difficulty that no mathematician has been able to obtain an accurate solution.

For simplicity it is now assumed that the depth of the arch ring is small compared with the radius of curvature. This means that two near right sections of the arch are practically parallel. With this assumption, Eq. 1 becomes strictly fundamental, for at any section we have a case of simple eccentric loading. As previously roughly outlined, the eccentric load is given by the pressure curve and force diagram.

Application of Eq. 1. Art. 11.

Lemma.—Eq. 1 shows that the difference between the gravity axis fiber stress and any other fiber stress distance c, equals $\frac{Mc}{I}$, the stress due to bending.

Let Fig. 5 represent a part of an arch under stress, The broken part represents the undeformed condition. Let l_c=length of any fiber distance c from the gravity axis. Before deformation, $l_c=l+c\phi$. By differentiation, $dl_c=dl+cd\phi$. . . . . . (a).

After deformation, $d(l_c+\Delta l_c)=d(l+\Delta l)+cd(\phi+\Delta\phi)$ . . (b), in which Δ means increment.

Equations (*a*) and (*b*) simply tell like facts about the two different conditions of the fiber.

(*b*)−(*a*) gives

$$d\Delta l_c = d\Delta l + c d\Delta\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e).$$

(*e*)÷(*a*) gives

$$\frac{d\Delta l_c}{dl_c} = \frac{d\Delta l + c d\Delta\phi}{dl + c d\phi} = \left(\frac{d\Delta l}{dl} + \frac{c d\Delta\phi}{dl}\right) \div \left(1 + c\frac{d\phi}{dl}\right)$$

$$= \left(\frac{d\Delta l}{dl} + \frac{c d\Delta\phi}{dl}\right) \div \left(1 + \frac{c}{k}\right),$$ in which k=rad. of curvature.

$= \frac{d\Delta l}{dl} + \frac{c d\Delta\phi}{dl}$, since c must be small compared with k.

By transpositions, $\frac{d\Delta l_c}{dl_c} - \frac{d\Delta l}{dl} = \frac{c d\Delta\phi}{dl}$ . . . (*f*).

Now $d(l+\Delta l) = dl + d\Delta l$, which shows that $d\Delta l$ is the deformation of dl when l becomes $l+\Delta l$ in the deformed fiber. Therefore, $\frac{d\Delta l}{dl}$ is the unit deformation of dl. Likewise $\frac{d\Delta l_c}{dl_c}$ of dl_c.

By Eq. (*f*), we see that $\frac{c d\Delta\phi}{dl}$ is the difference of the unit deformation of the gravity axis fiber, and a fiber distant c from the axis. Therefore from the foregoing lemma and mechanics of material,

$\frac{c d\Delta\phi}{dl} = \frac{Mc}{EI}$ · Integrating, $\Delta\phi = \frac{1}{E}\int\frac{Mdl}{I}$ . . . (*g*).

Eq. (*g*) gives the change of the angle between tangents at any two sections of the arch ring. Since the end tangents of a hingeless arch are fixed in direction, $\Delta\phi = o = \frac{1}{E}\int\frac{Mdl}{I}$, which leads directly to fundamental condition (2) in the first part of the work.

In the elaborate treatises of Prof. Howe and Prof. Dubois, the taking of c small compared with k is not done until it becomes absolutely necessary on account of the increased mathematical difficulties. We have made the assumption at the beginning, knowing by the previous view that it must be done. See Art. 10.

The simplicity of the above demonstration can be approached only by comparing it with the demonstration given in the aforesaid elaborate works.

Notice that $\frac{T}{A}$ does not affect equation (g) except as it affects the moment M. Remember, $M = Te$. See Fig. 3.

The remaining fundamental conditions involve x and y. The bending at the various sections not only changes the direction of the tangents, but the coordinates of the points also. Let Fig. 7 represent the gravity axis of the arch. Suppose one end is absolutely free. We will study the movement of this free end as caused by the stresses at any section. Consider any section, as at o. The effect of bending will be first considered. The bending will cause the free end to move from b to c. From similar triangles

$$\frac{bd}{bc}=\frac{y}{ob} \therefore bd=\frac{bc}{ob}y=y\Delta\phi.$$

Likewise $dc=\frac{bc}{ob}x=x\Delta\phi.$

The direct thrust, $\frac{T}{A}$, causes a uniform shortening at o in the direction of the tangent at that point. This shortening equals $T\frac{dl}{E}$, since we are only considering the internal stresses acting in one part of the arch, dl in length. This shortening can be resolved into two directions along the respective coordinates; or, bd is augmented by $\frac{Tdx}{AE}$ and dc by $\frac{Tdy}{AE}$. Hence for combined bending and direct thrust, the total movement of the free end along x is $y\Delta\phi+\frac{Tdx}{AE}$; and along y, it is $x\Delta\phi+\frac{Tdx}{AE}$. By integration, we sum up the total movement of the free end for all sections. Knowing that the movement must be zero, substituting the value of $\Delta\phi$ and neglecting $\frac{T}{A}$ for the time being, the fundamental conditions are $o=\int\frac{Mydl}{I}$ and $o=\int\frac{Mxdl}{I}$.

These conditions lead up to the fundamental equation (3) and (4) as explained in Art. 3.

Graphical Use of (2), (3) and (4). Art. 12.

The reasoning underlying the graphic application of (2), (3) and (4) will now be shown.

The following geometrical solutions are advanced. It is well to notice that the graphic difficulties encountered in the elastic theory, as applied to arches, are geometrical in character. Art. 11 covers most of the mechanics.

Given a series of parallel lines as in Fig. 8. Let *ABC* be a triangle covering the lines as shown. The triangle intercepts a portion of the parallel lines. Now if *A* moves along its line, the sum of the intercepted parts is a constant. Again, treat the intercepted parts as parallel forces. The center of gravity of the lines will lie in the same vertical for all positions of *A*.

Take *A* fixed, and *B* or *C* movable. The sum of the intercepted parts will now vary directly as the distance between *B* and *C*, but the center of gravity will remain in the same vertical.

The demonstration of these facts is left to the reader. These properties are true no matter what the shape of the triangle. Hence, it is convenient to use a right triangle in any problem involving the center of gravity of the parallel intercepted parts.

The foregoing simple properties are the very gist of the graphic method.

When the parallel lines are equidistant apart, the center of gravity can be located by a very simple formula. This is the reason, in Plate 2, the span was divided into equal spaces; and the formula, used for computing $v\,w$ in the location of $m\,m_1$, comes from this equal spacing.

In Fig. 9 given a curve—similar, say, to a trial equilibrium polygon—covering a series of parallel lines; required to draw a line, $m\,m_1$, fulfilling the requirements given in Part I. The reader should now re-read these requirements. To be general, the lines will be taken any distance apart, equal or unequal.

First the sum of the ordinates within the curve must be found, also their center of gravity found, regarding the ordinates as parallel forces. Call the sum R.

Suppose the line, $m\,m_1$, has been located. Since the sum of the ordinates, above the line to the curve, must equal the sum below, the sum of the ordinates between lines $m\,m_1$ and $v\,v_1$ must equal R.

Divide the quadrilateral into triangles as shown, and locate the centers of gravity of the ordinates within the triangles by using auxiliary right triangles as explained above, and shown in lower part of figure. The locations of the various centers of gravity are shown below the curve.

The sums of the ordinates, within the triangles composing the quadrilateral, are $\frac{Rc}{b+c}$ and $\frac{Rb}{b+c}$ respectively. These are shown in the figure. The sum of these sums, of course, equals R.

Let P_1 and P_2 be the sums of the ordinates within the upper and lower auxiliary right triangles respectively.

For simplicity the right triangles should have equal bases, say of length a.

Then keeping in mind the few facts set forth regarding a series of lines covered by a triangle, we can write $v\,m=\frac{aRc}{(b+c)P_1}$, and $v_1\,m_1=\frac{aRb}{(b+c)P_2}$. This locates the line $m\,m_1$.

If the ordinates are equidistant apart, or spaced symmetrically about the center of the arch, $P_1=P_2=\frac{a(n+1)}{2}$, in which n equals the number of spaces.

$$v\,m+v_1m_1=\frac{aRc}{(b+c)P_1}+\frac{aRb}{(b+c)P_2}=\frac{2R}{n+1},$$

$$\text{or } \frac{vm+v_1m_1}{2}=\frac{R}{n+1}.$$

This gives the center distance of the line $m\,m_1$ above $v\,v_1$.

Again, $v\,m-v_1m_1=\frac{2aR(c-b)}{(b+c)(n+1)a}=\frac{2M}{(b+c)(n+1)}$ in which $M=R(c-b)$, which is the moment of the ordinates within the curve, taken about the center of the arch.

The foregoing formulas enable us to locate $m\,m_1$ for all kinds of spacing. For equidistant spacing, the writer will leave the simplification of the formulas to the reader.* The simplified formulas are shown on Plate 2.

The general formulas must be used for unsymmetric arches, and in all cases where equidistant spacing is not adopted.

Prof. Wm. Cain † makes $\frac{dl}{I}$ a constant by dividing the gravity axis into short parts, such that $\frac{dl}{I}$ is the same for each part. This gives ordinates spaced symmetrically about the center, but not necessarily equally spaced. This is the only exact method for the investigation of arches which were not built to conform to equidistant spacing of the ordinates. A common case is the depth of ring a constant.

In designing a new arch, we can always use the artifice of equal spacing.

Referring now to Plate 2, the pressure curve, with the new pole distance and drawn through e_1 and e, is the original trial equilibrium polygon redrawn. The reader acquainted with the theory of the equilibrium polygon will understand this change.

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* See Transactions Am. Soc. of C. E., Vol. LV, p. 133.

† Steel Concrete Arches, No. 42 of Van Nostrand Science Series.

The new location of the trial equilibrium polygon makes the line, $m\,m_1$, coincide with line, $k\,k_1$. To keep the analogy, call ordinates, from $k\,k_1$ to the redrawn polygon, m^1 in value. When measured to the gravity axis, it was agreed to call them k ordinates in value.

Referring to the conditions underlying the use of Plate 2, at any ordinate, $t=m^1-k$; and for all ordinates, $\Sigma t=\Sigma m^1-\Sigma k=o$, since each term is zero by construction. This is condition (2).

For any ordinate, $tx=m^1x-kx$. For all ordinates, $\Sigma tx=\Sigma m^1x-\Sigma kx=o$ for the same reason as in previous paragraph. This is condition (3). Remember that in redrawing the trial equilibrium polygon with a new pole distance, the condition $\Sigma mx=o$ was not destroyed.

Notice that the above two conditions are true for any pole distance taken upon the horizontal line through r, or, to be general, for a line through r and parallel to $k\,k_1$.

Again, for any ordinate, $ty=m^1y-ky$. The moment, at the ordinate for the redrawn equilibrium polygon, equals m^1 (True Pole distance). For the trial equilibrium polygon, it is m (Trial Pole distance). By the well-known principle of the equilibrium polygon,

m (Trial Pole distance) $=m^1$ (True Pole distance). But the new pole distance was taken so that

$m^1 = m\frac{\Sigma ky}{\Sigma my}$. Therefore, $ty = my\frac{\Sigma ky}{\Sigma my} - ky$. For all ordinates, $\Sigma ty = \Sigma my\left(\frac{\Sigma ky}{\Sigma my}\right) - (\Sigma ky) = (1-1)\Sigma ky = o$. This is condition (4).

Temperature Stresses. Art. 13.

Conditions (2), (3) and (4) are true for any arch in which the horizontal thrust, H, is a constant. This is always the case for vertical loads.

Referring to Fig. 7 in which one end of the arch is taken free, any expansion or contraction will cause the free end to move.

Let T_o be the temperature at which the arch is not affected. This means the temperature at which it was built. T is any other temperature. D is the span, and f the coefficient of expansion. The movement of the free end is $(T-T_o)fD$. Now we can consider the arch as being forced into its real span through the distance, $(T-T_o)fD$, by an application of a horizontal force. This means that D really differs from the actual span by the amount of the movement, but it is close enough to take D equal to the actual span.

The horizontal force, wherever its point of application may be, is the same for all sections. Hence the fundamental conditions apply.

Notice that the hypothetical movement of the free end is along the axis of x only. Hence, applying an equation which leads up to one of our fundamental conditions,

$$D(T-T_o)f=\frac{Hdl}{EI}\int ty, \text{ or } H=\frac{I}{dl}D(T-T_o)f\frac{E}{\int ty}.$$

Since the end tangents are fixed in direction, $\Sigma t=o$. This means that H acts along the $k\,k_1$ line. H is now known in position and amount. Since the hypothetical movement is along the axis of x only, all of the conditions are not necessary.

For any temperature below T_0, H acts outwards; for a temperature above, inwards.

Notice that H varies directly as I. This means that great thickness of the arch ring means high temperature stresses. This is another strong argument for a shallow ring. We have already seen that the depth should be small compared with the radius of curvature. See Art. 10.

The range of temperature to be taken for designing, is an unsettled point. For the lower lake regions of this country, 50° F. is probably great enough. The normal temperature may be taken at 60° F. A fall of 35° F. and a rise of 15° F. will do.

Masonry arches covered with earth are not subject to extremes of heat.

The steel has been omitted in the above, but since the coefficient of expansion of steel and concrete are nearly the same, this is a very slight error.

The coefficient of expansion, f, equals 0.0000055.*

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* "Concrete, Plain and Reenforced," by Taylor and Thompson, p. 378.

The Effect of $\frac{T}{A}$ in Eq. 1. Art. 14.

The theory underlying the effect of $\frac{T}{A}$ is very difficult. Its effect can be shown only by a vigorous algebraic treatment, The student can find, in Prof. Howe's masterly treatise, such a treatment. The application of the theory is extremely long and tedious.

To overcome the great difficulties, Prof. Howe computed the actual effect on several arches, by algebraic processes; and compared the results gotten by neglecting $\frac{T}{A}$. He made a table in which the comparison is shown. In general, the effect of neglecting $\frac{T}{A}$ is to make H, the horizontal thrust or pole distance, too large.

For an arch having a rise of one-fourth the span, the true value of H is 93½% of the approximate value; for a rise of one-sixth, 86%; and for a rise of one-tenth, 69%. For intermediate rises, it will be close enough to interpolate.

Practically, $\frac{T}{A}$ can be neglected for all arches having a rise greater than one-fourth the span. For any rise less than this, take the true pole distance as calculated in Table 1. Multiply this pole distance by the percentage corresponding to the rise, and take this as the real true pole distance on the same line drawn from r. This will move the pole nearer the load line. With this revised pole distance, draw the pressure curve through the points e_1 and e. Remember that the points e_1 and e are not dependent on any particular pole distance so long as the pole is on the line drawn through r and parallel to $k\,k_1$, which means for a symmetric arch, horizontally.

Most writers neglect the effect of $\frac{T}{A}$. It is evident that for flat arches it is too important to be neglected. Many reenforced concrete arches are built quite flat.

The reader has now all of the theory necessary for a good understanding of the graphic method as applied to symmetric arches.

Some Objections to the Elastic Theory. Art 15.

One objection is its difficulty in theory and practice. This is largely the fault of writers. It is no more difficult than the theory of continuous beams, yet the theory of continuous beams furnishes the only practical determination of stresses in drawbridges. The difficulties can be largely overcome by the use of good English.

Another objection is that the external forces cannot be exactly determined. The most glaring case is an arch under a high embankment. A similar case is a lintel carrying a wall over a door or window. This objection is not peculiar to the elastic theory. It applies to any theory of the arch. It will not do to say that, since the forces cannot be determined, no calculation of stresses is necessary. The folly of this is evident in the broken lintels that can be often seen.

In railroad work, many arches are close to the track, and the loads are quite definite.

The exact determination of the points on the gravity axis at which the tangents are rigidly fixed is uncertain. The best that can be done, is to join the ring to the abutments by a rather sudden enlargement of section. In a two-hinged arch, this objection vanishes.

PART III.

UNSYMMETRIC ARCHES.

PART III.

UNSYMMETRIC ARCHES.

Unsymmetric Arches. Art. 16.

AN unsymmetric arch is one having abutments, or springings, on different levels.

The analysis of unsymmetric arches enables us to treat of arches acted upon by oblique forces.

Suppose we have a symmetric arch acted upon by parallel oblique forces. Conceive this arch to be tipped until the oblique forces become vertical, as in Plate 5. We now have an unsymmetric arch acted upon by vertical forces.

The artifice of tipping the arch enables as to analyze an arch for any set of oblique forces. Whatever the direction of the forces, they can be resolved in components which are vertical, and parallel to any oblique direction. The effect of the vertical components can be taken care of as shown in the previous pages. The method of dealing with the oblique components will be given in what follows. This double treatment will give a pressure curve for each set of components. At any section of the arch, the two pressure curves must be combined into a resultant.

An arched roof acted upon by wind loads and an arched dam in which the forces are radial, are cases requiring the investigation for oblique forces.

The writer will not give a detailed solution of a particular case. If the reader has grasped the principles in the foregoing pages, an outline of the method only is necessary.

In the previous pages the method of making $\frac{dl}{I}$ a constant was briefly outlined. It is now necessary to give a detailed exposition.

Let Fig. 10 represent a symmetric arch of any thickness of ring. Begin at the crown by assuming a short distance, dl, on the gravity axis. Compute I at the center of the part. Then $\frac{dl}{I}$ is determined for this part. Next, assume another short length, adjacent to the first part; and since the arch ring generally increases in thickness toward the springing, the dl for this part must be larger than for the part next to the crown. Continue with this mode of division until the springing is reached. The springing is represented by o and y on Plate 2. Several trials are necessary for each length dl.

If the arch ring is of constant depth, I is a constant; hence the dl length is the same for all parts.

On Plate 2 the depth of the arch ring was taken so that the division of the gravity axis gave equal spaces on the span. This artifice simplifies, wonderfully, the location of the pressure curve. This artifice can be used in most cases of design. In unsymmetric arches, it cannot be used. The more general method must be used.

In Plate 5 is shown the gravity axis of an unsymmetric arch. The division of the axis is supposed to have been made so that $\frac{dl}{I}$ is a constant. In this arch, the ordinates are best numbered from one end.

To make the case as general as possible, the ordinates locating the lower part of the curve at the lower springing, lap upon the ordinates of the upper part of the curve. These ordinates are shown by double lines.

A flat arch can be so tipped that no ordinates will overlap, but a semicircular arch cannot. The overlapping can be avoided by trying several different divisions of the gravity axis so that $\frac{dl}{I}$ is a different constant for each division, but this means considerable labor.

First, find the sum and center of gravity of all the ordinates to the gravity axis, remembering that the lapping ordinates are different in value but have the same moment arm for any origin of moments. The resultant is shown in position and amount on the drawing.

Secondly, use the artifice of two right triangles as explained in Part II. In finding the center of gravity of the ordinates within the triangles, the lapping ordinates are equal in value and have the same moment.

Next, treat the gravity axis curve the same as the trial equilibrium polygon was treated in Plate II, Fig. 9, locating the $k\ k_1$ line the same as the $m\ m^1$ line was there located. In an unsymmetric arch the $k\ k_1$ line is not parallel to the axis of x. The lapping ordinates present no further difficulties, since the centers of gravity were first determined.

The demonstrations given in Art. 11, must be kept clearly in mind.

Next, assume a force diagram and draw a trial equilibrium polygon, as shown below the arch. Locate the line, $m\ m_1$, the same as the $k\ k_1$ line was located. The right triangles, used for the $k\ k_1$ line, should be used for $m\ m_1$. The lapping ordinates must be considered as before.

Through o, the trial pole, draw $o\ r$ parallel to $m\ m_1$ line. From r draw a line parallel to the $k\ k_1$ line. The true pole is on this line. The true pole distance is found by the formula shown on Table I.

The points e_1 and e on the pressure curve, are found the same as for a symmetric arch.

In finding the $k\ y$ products at the double ordinates, the k and y ordinates, coming together at the respective points on the gravity axis, must be taken together.

In finding the $m y$ products at the double ordinates in the trial equilibrium polygon, the m ordinates of each pair are equal; but the double y ordinates are unequal. Hence, multiply each y ordinate by either one of the double ordinates at the corresponding point.

In general at every double ordinate, there are two $k y$ products and two $m y$ products.

The final pressure curve is drawn as shown. In drawing this curve the true pole distance was greatly exaggerated in order to get the curve within the limits of the drawing. This means that in measuring the ordinates from the $k k_1$ line to the pressure curve, we must take them diminished in the same proportion.

It must be remembered that for a perfectly elastic arch, the pressure curve may fall entirely out of the arch ring. Of course, it is absolutely necessary for the reader to understand the theory of eccentric stresses as given in any standard work on the mechanics of materials. In the consideration of temperature stresses, we had a case of this kind. There, the pressure curve was composed of a single horizontal force.

In analyzing an unsymmetric arch, a table similar to Table I must be made. Such a table was made in working up the plate, but is not presented here on account of the size. There is at best considerable labor in analyzing an arch for oblique forces.

In flat arches the horizontal forces can be neglected. The writer up to this time has neglected these forces; but for arches of considerable rise, say greater than one-fourth the span, he is now thinking of calculating for horizontal forces.

In arches having no lapping ordinates, the labor is considerably reduced. We took the worst case possible. This being understood, the simpler cases will present no difficulty.

PART IV.

APPENDIX.

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APPENDIX.

Some Vagaries in the Development of Reenforced Concrete. Art. 17.

LORD MACAULAY, years ago, said that if the law of universal gravitation had a pecuniary interest back of it, it would still be disputed.

A few years ago when reenforced concrete was brought before the public, it was stated that the imbedded steel conferred great stretching qualities on the surrounding concrete. Eminent authorities were quoted in proof of this. It is astonishing that any engineer should, for a moment, allow such a pseudo change of the elastic properties of material to become a reality to him. This vagary should be classified with the perpetual-motion kind. Prof. Turneaure's water marks and Prof. Talbot's fine experiments pricked this bubble.

The shrinkage of concrete is another subject which is much befogged. What a use of steel bars we would have if we allowed the sellers of steel bars to determine the amount of steel necessary! This question will be finally settled by experiments.

The use of bars having a high elastic limit is a vagary much dwelled upon by invested interests.

Since Talbot's and Turneure's experiments, it is pretty well settled that cracks, in the tension side of a beam, appear at a rather low stress in the steel. In view of this the writer fails to see why steel having an elastic limit of 30,000 lbs. is not just as good as steel having 60,000 lbs. The stretching quality of both kinds of steel is the same. But, some one will say, how about the ultimate strength of the beam? This brings us to the very gist of the matter.

One of the finest legacies possessed by the present generation of engineers is Hooke's Law. Were it not for this law, the science of mechanics of materials would be entirely different, if it existed at all. Designing would be largely rule of thumb.

Many experimenters are now trying to overthrow this law. Reenforced concrete beams are tested to destruction. The strength of the beam is calculated by a formula based on Hooke's Law, and found to disagree. Conclusion, Hooke's Law cannot be applied to reenforced concrete. Good logic, isn't it? They say the stress-strain curve is a parabola with the apex turned one way, or another way, or it is a hyperbola. But why can it not be a spiral, a cissoid, or a witch of Agnesi?

A curious mental aberration in the history of experiments, is the attempt to disprove the ordinary beam formula, founded on Hooke's law, by testing beams to destruction. This would be excusable had not such sane-minded writers as Merriman and Church, long ago, shown the exact limitations and correct applications of the beam formula.

Some years ago, a warm discussion was carried on, in a leading technical journal, about this formula. It is interesting to note that some bright engineers came out of the contest rather badly scarred.

The writer has before him a pamphlet which purports to instruct how to design a reenforced concrete. arch. The author of the pamphlet assumes the gravity axis to be a parabola. This allows the use of simple formulas for computing the various thrusts. The formulas are based on Hooke's law. In fact, the elastic theory is based on this law. But here is the curious part. When the author designs the various sections of the arch ring, he throws away Hooke's law, and uses the ultimate strength method.

Apparently, Hooke's law was right for determining the abutment reactions, but not for the sections. The pamphlet, evidently, has some invested interests back of it.

The ultimate strength method is a turning back to the days when engineering was not a learned profession.

Hooke's law, as applied to reenforced concrete beams, the reader may find set forth in Merriman's "Mechanics of Materials," and "Concrete, Plain and Reenforced," by Taylor and Thompson. Plate 3 is the drafting-room instructions issued by the writer to his draftsmen. This plate was gotten out before the above works were issued.

In testing materials the more heterogeneous the material the larger the specimen should be for testing. This simple fact is often overlooked.

In 1895 the Austrian Society of Civil Engineers tested full-sized arches. As a result, they found the elastic theory almost perfectly applicable to masonry arches.

So far no one has succeeded in applying the ultimate strength method to a case of combined thrust and moment. Yet, this is just the condition found in a reenforced concrete arch.

Graphical and Algebraic Methods. Art. 18.

The writer wishes to discuss, briefly, some processes in calculations.

Roughly, writers on technical subjects are divided into two classes, the graphical class and the algebraic class. Each makes a hobby of his method. Neither class is entirely right. It depends upon the nature of the problem to be solved.

Eddy's "Researches in Graphical Statics" and

Howe's Treatise on Arches, are the very opposite of each other. Both, to some extent, cover the same subject. The writer confesses that Eddy's book is the most difficult reading he ever tried. Professor Howe's is quite easy. Yet, Professor Howe's method is long and tedious in its application, while Professor Eddy's, as applied to arches, is short.

To Professor Eddy belongs the credit of applying the equilibrium polygon to the elastic arch. His method was simplified by Professor Wm. Cain, and is the basis of this work.

The graphical method is better in its solution of the arch, while the algebraic is better for a full development of the theory. Both are necessary for a full understanding and application of the subject. The algebraic method is the more powerful, as is seen in the discussion of the effect of $\frac{T}{A}$ in Professor Howe's work. This could hardly have been evolved by graphics.

Much of the popularity of graphics is based on a fallacy.

It is a well-known psychological law that the mind seems to understand better what the hand has been repeatedly taught to do. The result is that the average student takes readily to graphics. But no matter how much the muscular movement may help us, something else must be supplied before complete knowledge results. For instance, our lower jaw moves quite vigorously in eating, but that does not enable us to understand the mastication of food.

The writer has draftsmen who can apply the graphic method as outlined in Part I, but some of them do not understand the reasoning underlying it. The graphic method can be made a very poor educational process for the reason that it enables students to accomplish things without knowing why.

The writer believes that every graphical treatment of a subject should be accompanied by a thorough algebraic treatment, or at least the reason for every graphical step should be fully explained.

On the other hand, the educational defects of the graphical method, are its strong points in the drafting-room. In the drafting-room, not very much time can be given to explaining theory.

In graphics, lines must meet in a certain point or lie in certain directions. This enables a chief draftsman to check the work quickly. Algebraic methods cannot be so readily checked.

Each method has its use, and neither is exclusively better than the other.

Algebraic methods can often be made quite expeditious by proper tabulation, and by use of the slide rule.

A Drafting Room Definition of a Civil Engineer.
Art. 19.

Many definitions have been proposed for the term civil engineer. Some time ago the railroads of this country were required to fill out blanks giving the character, etc., of their employees. The blanks were to be sent to Washington city.

One employee, who had been carrying an engineer's tape on track work for about three months, filled his blank as a civil engineer. This service was really the extent of his ability.

The writer proposes the following definition from the drafting-room standpoint. The definition can be divided into two parts, an ethical part and an intellectual part.

Ethical Part.—An engineer is a man of rigid honesty. He is too honest to allow a steel bar company to design his reenforced concrete work.

Intellectual Part.—An engineer is a man who can read Church's "Mechanics of Engineering," or any other standard work.

The two parts are closely connected. A deficiency in the second part is often accompanied by a deficiency in the first part. Stupidity and dishonesty are nearly always together.

Hoskins' Hydraulics.

By L. M. HOSKINS, Professor in Leland Stanford University. 8vo. 271 pp. $2.50.

A comprehensive text-book, intended for the fundamental course in the subject usually offered in schools of engineering, but somewhat more compact in treatment than the ordinary treatise now available.

In the presentation of working rules for estimating flow in the various practical cases met by the engineer the aim of the book is to give a clear statement of the rational basis of the formula adopted, and also to make clear to what extent the theory is defective and the formula therefore empirical. The author tries in all cases to avoid the appearance of precise knowledge where the reality is absent.

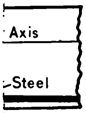
Benjamin's Machine Design.

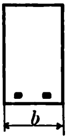
By CHARLES H. BENJAMIN, Professor in the Case School of Science. 12mo. 202 pp.

An enlargement of the author's previous little book on the subject, which aimed to give the essentials in compact shape. In its present form the book is still concise, but is better suited to text-book use. The subject-matter of the book is well up-to-date.

HENRY HOLT AND COMPANY
PUBLISHERS NEW YORK

TE 3
R STEEL-CONCRETE BEAMS





af=Total Tension in steel

e $\frac{cbhx}{2}$=" comp. " concrete

Tension in concrete
not considered

0,000,000 per □"

—varies

d at 10, 15, 20 and 30

$$\frac{Es}{Ec}(1-x)=\frac{x^2}{2p} \quad (3)$$

, then find x from table. Having x,
aving x and f, find c, stress in con-
cinder concrete.

x		
$Es \div Ec$		
5	20	30
.158	0.181	0.217
.217	0.246	0.292
.258	0.292	0.344
.292	0.328	0.384
.320	0.358	0.418
.344	0.384	0.446
.365	0.407	0.471
.384	0.428	0.493
.402	0.446	0.513
.418	0.463	0.531
.446	0.493	0.562
.471	0.519	0.588
.493	0.542	0.611
.513	0.562	0.631
.531	0.580	0.649

B.R. Leffler 8-31-05

4

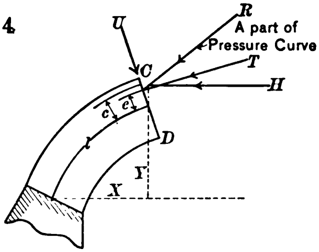
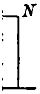


FIG.3



.6

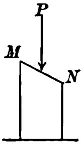
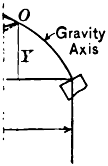


FIG.6a



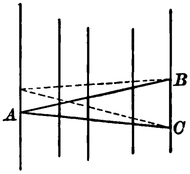


FIG.8

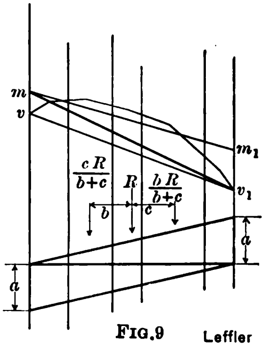


FIG.9 Leffler